Engineering Notes

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Attitude Motion of a Roll Accelerated Vehicle

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Nomenclature

$\mathbf{T}$ = applied torque vector
$\mathbf{h}$ = angular momentum vector
$\mathbf{\Omega}$ = angular velocity vector
p, q, r = angular velocity components
A, B = moment of inertia components
L, M = applied torque components
ϕ = roll angle
α = L/A, roll acceleration
γ = A/B, inertia ratio
a = M/B, transverse acceleration
ζ = $q + ir$, complex angular rate
x = $[\alpha(1-\gamma)/\pi]^{1/2} t$, generalized time
k = $\gamma/(1-\gamma)$
λ = $a[\pi/\alpha(1-\gamma)]^{1/2}$
ξ = complex attitude excursion
θ = pitch attitude
ψ = yaw attitude
t = time

Introduction

WHEN deploying a payload from a space satellite or launching a small rocket, the engineer is concerned with the accuracy with which this can be done. It is common practice to roll the deployed vehicle, while it is thrusting away from its initial position, in an attempt to average out disturbance torques and maintain attitude accuracy. This Note presents an analytic solution for the steady state attitude error for a constant roll accelerated symmetrical vehicle subjected to a constant body-fixed overturning (transverse) torque. A solution for the envelope of the attitude motion is also presented, and comparisons are made with numerical solutions. The significance of the analytic solution is paramount in preliminary design studies since time-consuming digital computer runs are eliminated.

Analysis

The coordinate system used in the following analysis is allowed to pitch and yaw, but not roll. This is convenient for symmetrical vehicles since the attitude motion may be visualized as the path traced out by the nose on a plane normal to the original attitude vector. Expanding the moment equation

$$\mathbf{T} = \dot{\mathbf{h}} + \mathbf{\Omega} \times \mathbf{h} \qquad (1)$$

gives the following set of equations:

$$A\dot{p} = L \qquad (2)$$

$$B\dot{q} + Apr = M \cos\phi \qquad (3)$$

$$B\dot{r} - Apq = M \sin\phi \qquad (4)$$

The overturning torque is arbitrarily assumed to be initially about the pitch axis. For simplicity, the initial vehicle rates are assumed to be zero; hence, the roll history is given by:

$$\phi = (\alpha/2)t^2 \qquad (5)$$

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Equations (3) and (4) are combined in the complex plane and integrated to give:

$$\zeta = ae^{i\gamma\phi} \int_0^t e^{i(1-\gamma)\phi}\, dt \qquad (6)$$

Making the substitution

$$(1-\gamma)\phi = (\pi/2)x^2 \qquad (7)$$

and changing the independent variable from t to x, Eq. (6) becomes:

$$\zeta = \lambda\, e^{ik(\pi/2)x^2} \int_0^y e^{i(\pi/2)x^2}\, dx \qquad (8)$$

where the integral term is recognized to be the complex Fresnel integral. The pitch and yaw rates are, respectively,

$$q = \lambda\{C(y)\cos[k(\pi/2)y^2] - S(y)\sin[k(\pi/2)y^2]\} \qquad (9)$$

$$r = \lambda\{S(y)\cos[k(\pi/2)y^2] + C(y)\sin[k(\pi/2)y^2]\} \qquad (10)$$

where

$$C(y) = \int_0^y \cos[(\pi/2)x^2]\, dx \qquad (11)$$

$$S(y) = \int_0^y \sin[(\pi/2)x^2]\, dx \qquad (12)$$

Assuming small attitude excursions and recalling our nonrolling coordinate system, the vehicle attitude is found by integrating Eq. (8).

$$\xi = \lambda \int_0^x e^{ik(\pi/2)y^2} \int_o^y e^{i(\pi/2)z^2}\, dz\, dy \qquad (13)$$

It can be shown from Ref. 1, that as $x \to \infty$, the integral in Eq. (13) becomes:

$$\xi = i(\lambda/\pi)[(1-\gamma)/\gamma]^{1/2} \tan^{-1}[(1-\gamma)/\gamma]^{1/2} \qquad (14)$$

The steady-state pitch and yaw attitudes are, respectively,

$$\theta = 0 \qquad (15)$$

$$\psi = (\lambda/\pi)[(1-\gamma)/\gamma]^{1/2} \tan^{-1}[(1-\gamma)/\gamma]^{1/2} \qquad (16)$$

Equation (15) is zero because we assumed the overturning torque to be initially about the pitch axis. If the yaw attitude is normalized by λ (which is a function of the spin acceleration, the overturning acceleration, and the inertia ratio); it is observed that this steady state error depends only on the inertia ratio. Figure 1 illustrates the normalized yaw history for typical inertia ratios.

Figure 1 indicates an oscillation about the steady state solution. In order to estimate the accuracy of where the vehicle is pointing, it is desirable to define the oscillation envelope.

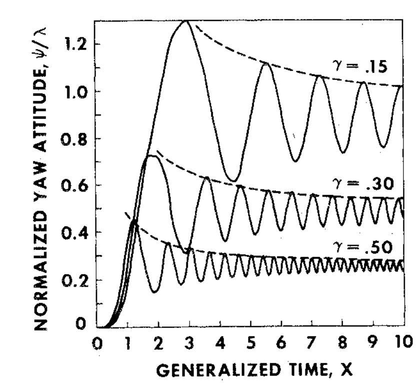

Fig. 1 Normalized yaw attitude.

For large values of x, the Fresnel integrals may be approximated by:

$$\int_0^x \sin[k(\pi/2)y^2]\,dy \doteq 1/2(k)^{1/2} - (\pi kx)^{-1}\cos[k(\pi/2)x^2] \tag{17}$$

$$\int_0^x \cos[k(\pi/2)y^2]]\,dy \doteq 1/2(k)^{1/2} + (\pi kx)^{-1}\sin[k(\pi/2)x^2] \tag{18}$$

If Eq. (13) is normalized by λ, integrated from x large to infinity, expanded into its real and imaginary components, it may be expressed as:

$$[\xi(x)-\xi(\infty)]/\lambda = (1/(2)^{1/2}\pi kx)\{\sin[k(\pi/2)x^2+\pi/4] - i\cos[k(\pi/2)x^2+\pi/4]\} \tag{19}$$

The amplitude of the oscillation is recognized to be $1/(2)^{1/2}\pi kx$. This formula is only true for large values of x; however, Fig. 1 indicates very good agreement even for small values of x.

Concluding Remarks

This Note has presented a simple closed-form expression for determination of the final attitude dispersions of a symmetrical roll accelerated vehicle subjected to body fixed transverse torque. In addition, an expression describing the oscillatory envelope is also given. Comparison of the closed-form solutions with numerical integration of the equations indicates good agreement; hence, the closed-form solutions give the analyst a quick and accurate means of evaluating this problem.

References

[1] Rosser, J. B., *Theory and Application of* $\int_0^\xi e^{-x^2}dx$ *and* $\int_0^\xi e^{-p^2y^2}dy\int_0^y e^{-x^2}dx$, Mapleton House, New York, 1948, pp. 102–105.

Suboptimal Controller for a Linearized n-Body Spacecraft

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Nomenclature

B	= diagonal damping matrix
$DEZ\{\ \}$	= vector deadzone function
E	= unit (identity) matrix
i	= integer used to designate a joint
k	= diagonal stiffness matrix
n	= number of rigid bodies in spacecraft model
r	= number of degrees of freedom
α	= matrix used in state equations [see Eq. (1)]
γ_k, γ	= relative angular motion at joint k; γ is $r-3\times 1$ vector having components γ_k, $k=1,2,\ldots,r-3$
θ_i, ω_{0_i}	= attitude angles of base body 0 (components of vector θ); angular velocity measure numbers of base body 0, $i=1,2,3$ (components of vector ω_0)
$\omega_0, \omega_R, \omega$	= angular velocity vector of base body 0; relative angular velocity components $\dot\gamma_k$, $k=1,2,\ldots,r-3$; ω has components ω_0 and ω_R
$A_{11}, A_{12}, A_{21}, A_{22}$	= elements of partitioned matrix appearing in Eq. (1)
b_i	= basis vectors for base body 0, $i=1,2,3$
B_i	= damping coefficients for joint i
$C_\theta, C_{\omega 0}, C_\gamma, C_{\omega R}$	= control gain matrices
$F_{\lambda_j}, M_{\lambda_j}$	= interaction force and moment on body λ due to joint j
F_λ, M_λ	= vectors representing externally applied forces and moments to body λ
K_i	= stiffness coefficients for joint i
L_0, L_R, L	= vector forcing function for base body; vector forcing function for $n-1$ remaining bodies; vector L has components L_0 and L_R
$\bar L_0, \bar L_R$	= vector forcing functions used in defining L; $\bar L_0$ and $\bar L_R$ are formed from L_0 and L_R by not including the terms $K\gamma + B\dot\gamma$
L_{00}, L_{0R}	= vector forcing functions used in defining $\bar L_0$; L_{00} is the contribution to $\bar L_0$ due to forces F_0 and moments M_0 applied to the base body; L_{0R} is the contribution made by forces F_λ and moments M_λ with $\lambda \neq 0$
L_{R0}, L_{RR}	= vector forcing functions used in defining $\bar L_R$; L_{R0} is the contribution to $\bar L_R$ due to F_0; L_{RR} is the contribution made by F_λ and M_λ with $\lambda \neq 0$
$\bar L_{00}$	= vector forcing function used in defining L_{00}. $\bar L_{00}$ is formed from L_{00} by not including the moment M_0
u_0, u_R	= suboptimal control vector for base body 0; suboptimal control vector for $n-1$ remaining bodies

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Introduction

FOR deep-space missions, the requirements placed on antenna pointing, articulation control, science platform settling times, etc., tend to become continually more stringent. Moreover, to meet the objectives of the scientific experiments and to provide isolation from the radiation produced by the power source, booms are frequently employed. These facts dictate that a suitable spacecraft model must be determined and, in addition, that sensor noise and plant disturbances be accounted for.

Considerable attention, in the open literature, has been focused on the problem of developing a suitable set of deterministic dynamical equations for a spacecraft.[1–6] Recently, a particularly elegant albeit complicated set of dynamical equations for an n-hinged rigid-body spacecraft has been developed.[2] The salient features of this set of dynamical equations are that 1) constraint torques do not appear, and 2) the number of variables involved is equal to the number of degrees of freedom of the system. Stochastic control theory has also been given special attention in the open literature. Reference 7 is devoted exclusively to linear stochastic optimal control of linear systems subject to the expected value of a quadratic cost function.

In the present work, the objective is to determine a controller which makes use of the elaborate, deterministic model of the spacecraft and, in addition, accounts for sensor noise, disturbances, etc. In essence, an optimal stochastic controller is sought. However, because of the practical importance of ease of implementation, simplicity, and reliability, a suboptimal stochastic controller is determined.

It is known that a solution can be found to a linear stochastic optimization problem involving a quadratic cost functional. However, the plant representing the dynamics of the spacecraft is nonlinear. In addition, a single quadratic cost function which accounts for *all* of the desired characteristics of the controller *cannot* be found. Nevertheless, a suboptimal stochastic controller is obtained by: 1) appropriately linearizing the dynamical model of the spacecraft to obtain the plant; 2) invoking the "certainty-equivalence" principle of modern control theory to arrive at the structure of the optimal linear controller; 3) generating the suboptimal controller.

The contributions of this paper include the casting of the dynamical equations for the spacecraft in a form suitable for optimal stochastic control theory and the development of a suboptimal stochastic controller. To the writer's knowledge, a stochastic controller based on a realistic model of a complex spacecraft has not been previously obtained.